

FINAL TERM EXAMINATION 2009

Calculus & Analytical Geometry-I

Time: 120 min

Marks: 80

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) > 0$ then f has relative At x_0

- ▶ Minima
- ▶ Maxima
- ▶ None of these

Question No: 2 (Marks: 1) - Please choose one

In the notation
 $\int f(x)dx = F(x) + C$
C represents

- ▶ A polynomial
- ▶ A Constant
- ▶ A Variable
- ▶ None of these

Question No: 3 (Marks: 1) - Please choose one

According to Power-Rule of differentiation, if $f(x) = x^n$ where n is a real number, then

$$\frac{d}{dx}[x^n] =$$

- ▶ x^{n-1}
- ▶ $n x^{n-1}$
- ▶ $n x^{n+1}$

▶ $(n-1)x^{n+1}$

Question No: 4 (Marks: 1) - Please choose one

If $2x - y = -3$ then $\frac{dy}{dx} =$

- ▶ 2
- ▶ -2
- ▶ 0
- ▶ -3

Question No: 5 (Marks: 1) - Please choose one

$30^0 =$ _____

▶ $\frac{\pi}{3}$

▶ $\frac{\pi}{4}$

▶ $\frac{\pi}{6}$

▶ $\frac{\pi}{2}$

Question No: 6 (Marks: 1) - Please choose one

If a function g is differentiable at a point x and a function f is differentiable at a point $g(x)$, then the _____ is differentiable at point x .

- ▶ Composition ($f \circ g$)
- ▶ Quotient (f / g)
- ▶ Product ($f \cdot g$)
- ▶ Sum ($f + g$)

Question No: 7 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denote points in that

interval. If $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ then which of the following statement is correct?

- ▶ f is an increasing function.
- ▶ f is a decreasing function.
- ▶ f is a constant function.

Question No: 8 (Marks: 1) - Please choose one

If $f''(x) < 0$ on an open interval (a,b) then which of the following statement is correct?

- ▶ f is concave up on (a, b).
- ▶ f is concave down on (a, b)
- ▶ f is linear on (a, b).

Question No: 9 (Marks: 1) - Please choose one

$$\sum_{k=1}^n f(x_k^*) \Delta x_k$$

The sum is known as:

- ▶ Riemann Sum
- ▶ General Sum
- ▶ Integral Sum
- ▶ Geometric Sum

Question No: 10 (Marks: 1) - Please choose one

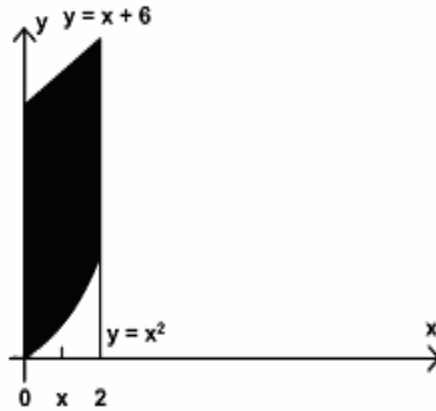
$$\sum_{k=1}^n f(x_k^*) \Delta x_k$$

What does 'n' represent in Riemann Sum ?

- ▶ No. of Circles
- ▶ No. of Rectangles
- ▶ No. of Loops
- ▶ No. of Squares

Question No: 11 (Marks: 1) - Please choose one

What is the area of the region in the following figure?



▶ $A = \int_0^2 [(x+6) - (x^2)] dx$

▶ $A = \int_x^2 [(x+6) - (x^2)] dx$

▶

▶ $A = \int_0^2 [(x+6) + (x^2)] dx$

▶

▶ $A = \int_0^x [(x+6) - (x^2)] dx$

▶

Question No: 12 (Marks: 1) - Please choose one

If $\int_1^4 f(x) dx = 2$ and $\int_1^4 g(x) dx = 10$ then which of the following is value of

$\int_1^4 [3f(x) - g(x)] dx$?

- ▶ 16
- ▶ 12
- ▶ -4
- ▶ -8

Question No: 13 (Marks: 1) - Please choose one

$\int_0^1 2x(x^2 + 4) dx =$ _____

$\frac{9}{2}$

$\frac{5}{2}$

$\frac{2}{5}$

$-\frac{9}{2}$

Question No: 14 (Marks: 1) - Please choose one

Let f is a smooth function on $[0, 3]$. What will be the arc length L of the curve $y = f(x)$ from $x = 0$ to $x = 3$?

$L = \int_0^3 \sqrt{1 + [f(x)]^2} dy$

$L = \int_a^b \sqrt{1 + [f'(x)]^2}$

$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dy$

$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dx$

Question No: 15 (Marks: 1) - Please choose one

Let f be a smooth, nonnegative function on $[1, 3]$. What is the surface area S generated by revolving the portion of the curve $y = f(x)$ between $x = 1$ and $x = 3$ about the x -axis?

$$S = \int_0^2 2\sqrt{1+[f(x)]}dx$$



$$S = \int_0^3 2\pi f(x)\sqrt{1+[f'(x)]}dx$$



$$S = \int_0^2 2\sqrt{1+[f'(x)]}dx$$



$$S = \int_1^3 2\pi f(x)\sqrt{1+[f'(x)]^2} dx$$



Question No: 16 (Marks: 1) - Please choose one

Let an object is displaced 2m by a force of 2N. What is the work done W?

▶ - 4

▶ 4

▶ 2

▶ 0

Question No: 17 (Marks: 1) - Please choose one

$$\int_a^{+\infty} f(x)dx = \lim_{l \rightarrow \infty} \int_a^l f(x)dx$$

Consider the improper integral
which of the following can be occurred?

if the limit exists then

▶ Diverges

▶ Converges

▶ Test fail

Question No: 18 (Marks: 1) - Please choose one

If f is continuous on (a, b] but does not have a limit from the right then the integral

$$\int_a^b f(x)dx = \lim_{l \rightarrow a} \int_l^b f(x)dx$$

defined by

is called :

▶ Improper

- ▶ Proper
- ▶ Line

Question No: 19 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the difference between successive terms $a_{n+1} - a_n < 0$ then the sequence is known as :

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing

Question No: 20 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} > 1$ then the sequence is known as:

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing

Question No: 21 (Marks: 1) - Please choose one

Which of the following is true for the sequence $\{n\}_{n=0}^{\infty}$?

- ▶ Nonincreasing
- ▶ Nondecreasing
- ▶ Increasing
- ▶ Decreasing

Question No: 22 (Marks: 1) - Please choose one

If $f(n) = a_n$ is the nth term of the sequence and f is differentiable and $f'(n) \leq 0$ then the sequence will be :

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing

► Nonincreasing

Question No: 23 (Marks: 1) - Please choose one

If Newton's Method is used to approximate the real solutions of the equation

$x^3 + x - 3 = 0$ and the first guess $x_1 = 1$, What is x_2 ?

► $\frac{5}{4}$

► $\frac{1}{4}$

► $\frac{3}{4}$

► $\frac{-1}{2}$

► $\frac{3}{2}$

► $\frac{3}{4}$

► $\frac{3}{2}$

► $\frac{3}{2}$

► $\frac{3}{2}$

Question No: 24 (Marks: 1) - Please choose one

Suppose that we apply Newton's Method to approximate the real solutions of the

equation $x^3 - 2x^2 - 1 = 0$. If we start at $x_1 = 2$, then which of the following is value of x_2 ?

► 6

► 2.25

► 0

► 2

Question No: 25 (Marks: 1) - Please choose one

If the sequence of partial sum of a series converges then what will the series show itself ?

► Diverges

► Converges

► Gives no information

Question No: 26 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k \rightarrow \infty} \frac{u_{k+1}}{u_k}$ if $\rho > 1$, then which of the following is true?

- ▶ Converges
- ▶ Diverges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 27 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k \rightarrow \infty} \frac{u_{k+1}}{u_k}$ if $\rho = 1$, then which of the following is true?

- ▶ Converges
- ▶ Diverges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 28 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k \rightarrow \infty} \sqrt[k]{u_k} = \lim_{k \rightarrow \infty} (u_k)^{\frac{1}{k}}$ if $\rho = 1$, then which of the following is true?

- ▶ Converges
- ▶ Diverges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 29 (Marks: 1) - Please choose one

For an alternating series to be convergent which of the following condition must be satisfied?

- ▶ $\lim_{k \rightarrow \infty} a_k = 1$
- ▶ $a_1 > a_2 > a_3 \dots > a_k > \dots$
- ▶ $a_1 \leq a_2 \leq a_3 \dots \leq a_k \leq \dots$
- ▶ Gives no information

Question No: 30 (Marks: 1) - Please choose one

For an alternating series to be convergent which of the following condition must be satisfied?

▶ $a_1 \geq a_2 \geq a_3 \dots \geq a_k \geq \dots$

▶ $\lim_{k \rightarrow \infty} a_k = 0$

▶ $a_1 \leq a_2 \leq a_3 \dots \leq a_k \leq \dots$

▶ $\lim_{k \rightarrow \infty} a_k = 1$

▶

Question No: 31 (Marks: 1) - Please choose one

What is the base of natural logarithm?

▶ 2.71

▶ 10

▶ 5

▶ Any real number

Question No: 32 (Marks: 1) - Please choose one

A function F is called an antiderivative of a function f on a given interval if _____ = $f(x)$, for all x in that interval.

▶ $F'(x)$

▶ $F(x)$

▶ $f'(x)$

▶ $f''(x)$

▶

Question No: 33 (Marks: 1) - Please choose one

$\log_b ac =$ _____

▶ $\log_b a + \log_b c$

▶ $\log_b a - \log_b c$

▶ $\frac{\log_b a}{\log_b c}$

▶ $(\log_b a)(\log_b c)$

Question No: 34 (Marks: 1) - Please choose one

$\log_b a^r =$ _____

▶ $a \log_b r$

▶ $r \log_b a$

▶ $\frac{\log_b a}{\log_b r}$

▶ $\log_b a + \log_b r$

Question No: 35 (Marks: 1) - Please choose one

$\log_b \frac{1}{c} =$ _____

▶ $\log_b c$

▶ $1 - \log_b c$

▶ $-\log_b c$

▶ $1 + \log_b c$

Question No: 36 (Marks: 1) - Please choose one

$$\log_b \frac{1}{t} = \underline{\hspace{2cm}}$$

- ▶ $\log_b t$
- ▶ $1 - \log_b t$
- ▶ $1 + \log_b t$
- ▶ $-\log_b t$

Question No: 37 (Marks: 1) - Please choose one

What is the sum of following series?

$$1^2 + 2^2 + 3^2 + 4^2 + \text{-----} + n^2$$

- ▶ $\frac{n(n+1)(2n+1)}{6}$
- ▶ $\frac{n(2n)(2n+1)}{6}$
- ▶ $\frac{(n+1)(n+2)}{2}$
- ▶ $\frac{(n+1)(2n+1)}{6}$
- ▶

Question No: 38 (Marks: 1) - Please choose one

$$\sum_{k=1}^n \frac{k^3}{2} = \underline{\hspace{2cm}}$$

$$\frac{n(n+1)}{4}$$



$$\frac{[n(n+1)]^2}{8}$$



$$\frac{n(n+1)(2n+1)}{12}$$



Question No: 39 (Marks: 1) - Please choose one

$$y = \frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x; 0 \leq x \leq 1$$

Let _____ then which of the following is the length of the curve?

$$L = \int_0^1 \sqrt{1 + \left[\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x \right) \right]^2} dx$$



$$L = \int_0^1 \sqrt{\left[\frac{d}{dx} \left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x \right) \right]^2} dx$$



$$L = \int \sqrt{1 + \left[\frac{d}{dx} \left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x \right) \right]^2} dx$$



$$L = \int_0^1 \sqrt{1 + \left[\frac{d}{dx} \left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x \right) \right]^2} dx$$



Question No: 40 (Marks: 1) - Please choose one

If $f(x) = e^{-x}$ at $x = 0$ be the Taylor series, then which of the following is also true?

- ▶ Arithmetic series
- ▶ Maclaurin series
- ▶ Geometric series
- ▶ Harmonic series

Question No: 41 (Marks: 2)

$$u = \frac{\pi}{2} - x \qquad \int_0^{\pi} \sin\left(\frac{\pi}{2} - x\right) dx$$

Using substitution transform the integral into variable u.

Question No: 42 (Marks: 2)

$$\int_3^{+\infty} \frac{dx}{2x^2}$$

Evaluate the improper integral

Question No: 43 (Marks: 2)

A function $f(x) = 6 - 2x - x^2$ has critical point 1 in an interval $[-4, 3]$. Find the absolute minimum value of the function.

Question No: 44 (Marks: 3)

Find the absolute maximum value of the function:

$$f(x) = 2x^3 + 3x^2 - 12x + 4 \quad \text{on} \quad [-4, 2]$$

Question No: 45 (Marks: 3)

Find the area of the region bounded by the curve $y = x^2 - 4x - 5$ and $y = x + 1$ (do not evaluate).

Question No: 46 (Marks: 3)

$$\left\{ \frac{3}{n^2} \right\}_{n=5}^{\infty}$$

Determine whether the following sequence is strictly monotone:

Question No: 47 (Marks: 5)

Determine whether the sequence converges or diverges. If converges find limit

$$\lim_{n \rightarrow \infty} \frac{3^n + (-1)^n}{3^{n+1} + (-1)^{n+1}}$$

Question No: 48 (Marks: 5)

Find the lengths of the curves

$$x = \frac{t^2}{2}, \quad y = \frac{(2t+1)^{\frac{3}{2}}}{3}, \quad 0 \leq t \leq 4$$

Question No: 49 (Marks: 5)

Evaluate the indefinite integral $\int [(x^4 + 2)] [\cos(x^5 + 10x)] dx$ by substitution method.

Question No: 50 (Marks: 10)

Find the Maclaurin series for $f(x) = e^{2x}$