MTH501 Final Paper 07 Feb 2012 by SHINING STAR

1. Find the distance

SOLUTION:

$$x.y = \begin{bmatrix} 10-(-1) \\ -3-(5) \end{bmatrix}$$

$$x.y = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$dis(x.y) = \|x-y\| = \sqrt{(11)^2+(2)^2}$$

$$= \sqrt{121+4}$$

$$= \sqrt{125}$$

2. Find the eigen values and corresponding eigen vectors of a matrix A = $\begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix}$

SOLUTION:

$$Ax = \lambda x$$

$$(Ax-\lambda x) = 0$$

$$(A-\lambda I)x = 0$$

$$For \lambda = 1$$

$$\begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -0.05 & 0.03 \\ 0.05 & -0.03 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-0.05x_1 + 0.03x_2 = 0$$

$$x_1 = \frac{0.03}{0.05}x_2 \quad or \quad x_1 = \frac{3}{5}x_2$$

in parametric form

$$x_1 = \frac{3}{5}t \text{ and } x_2 = t$$

3. SHOW that y and z is orthogonal

SOLUTION:

$$y.z = \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix} . \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$$

$$y.z = \begin{bmatrix} -3 \\ -56 \\ 60 \\ 0 \end{bmatrix}$$

$$\boldsymbol{u_1} = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \boldsymbol{u_2} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \boldsymbol{u_3} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \text{and } \boldsymbol{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$

4. Let Show that $\{\boldsymbol{u_1}, \boldsymbol{u_2}\}$ or $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ is an orthogonal basis for $\boldsymbol{R^2}$ and $\boldsymbol{R^3}$, respectively.

SOLUTION:

We can write it as

$$x = c_1u_1 + c_2u_2 + c_3u_3$$

$$c_1 = \frac{x.u_1}{u_1.u_1} = \frac{\begin{bmatrix}5\\-3\\1\end{bmatrix}.\begin{bmatrix}3\\-3\\0\end{bmatrix}}{\begin{bmatrix}3\\-3\\0\end{bmatrix}.\begin{bmatrix}3\\-3\\0\end{bmatrix}} = \frac{15+9+0}{9+9+0} = \frac{24}{18} = \frac{12}{9} = \frac{4}{3}$$

$$c_2 = \frac{x.u_2}{u_2.u_2} = \frac{\begin{bmatrix}5\\-3\\1\end{bmatrix}.\begin{bmatrix}2\\2\\-1\end{bmatrix}}{\begin{bmatrix}2\\2\\-1\end{bmatrix}.\begin{bmatrix}2\\2\\-1\end{bmatrix}} = \frac{10-6-1}{4+4+1} = \frac{3}{9} = \frac{1}{3}$$

$$c_3 = \frac{x.u_3}{u_3.u_3} = \frac{\begin{bmatrix}5\\-3\\1\end{bmatrix}.\begin{bmatrix}1\\1\\4\end{bmatrix}}{\begin{bmatrix}1\\1\\4\end{bmatrix}.\begin{bmatrix}1\\1\\4\end{bmatrix}} = \frac{5-3+4}{1+1+8} = \frac{6}{10} = \frac{3}{5}$$

$$x = \frac{4}{3}u_1 + \frac{1}{3}u_2 + \frac{3}{5}u_3$$

$$\hat{x} = \begin{bmatrix}1\\1\end{bmatrix}$$

5. Compute the least square error associated with the least square solution of the

$$A = \begin{bmatrix}1 & 3\\1 & -1\\1 & 1\end{bmatrix}, b = \begin{bmatrix}5\\1\\0\end{bmatrix}$$

equation $Ax = b$ where

6. Solve the following homogeneous system of linear equations:

$$\begin{aligned} x_1 + 3x_2 - x_3 &= 0 \\ x_2 - 8x_3 &= 0 \\ 4x_3 &= 0 \end{aligned}$$

7. Solve the initial value problem $x'(t) = Ax(t)$ for $t \geq 0$, with $x(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Where $A = \begin{bmatrix} -2 & -5 \\ 1 & 4 \end{bmatrix}$, with eigenvectors $v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$

8. Let V be in inner product space. Show that if u and v are orthogonal vectors in V such that $\|u\| = \|v\| = 1$, then $\|u - v\| = \sqrt{2}$.