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Virtual University of Pakistan

MTH501 Final Paper 07 Feb 2012 by  
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1. Find the distance

**SOLUTION:**

$$x.y = \begin{bmatrix} 10 - (-1) \\ -3 - (5) \end{bmatrix}$$

$$x.y = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$\begin{aligned} \text{dis}(x.y) &= |x - y| = \sqrt{(11)^2 + (2)^2} \\ &= \sqrt{121 + 4} \\ &= \sqrt{125} \end{aligned}$$

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$$\begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix}$$

2. Find the eigen values and corresponding eigen vectors of a matrix A =

**SOLUTION:**

$$Ax = \lambda x$$

$$(Ax - \lambda x) = 0$$

$$(A - \lambda I)x = 0$$

$$\text{For } \lambda = 1$$

$$\begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -0.05 & 0.03 \\ 0.05 & -0.03 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-0.05x_1 + 0.03x_2 = 0$$

$$x_1 = \frac{0.03}{0.05}x_2 \text{ or } x_1 = \frac{3}{5}x_2$$

in parametric form

$$x_1 = \frac{3}{5}t \text{ and } x_2 = t$$

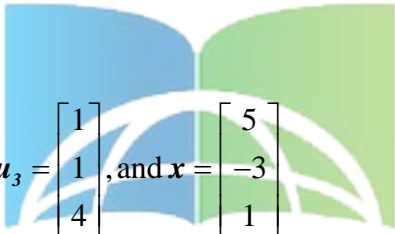
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3. SHOW that y and z is orthogonal

**SOLUTION:**

$$y \cdot z = \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$$

$$y \cdot z = \begin{bmatrix} -3 \\ -56 \\ 60 \\ 0 \end{bmatrix}$$


$$\mathbf{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \text{ and } \mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$

4. Let  $\{\mathbf{u}_1, \mathbf{u}_2\}$  or  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is an orthogonal basis for  $\mathbf{R}^2$  and  $\mathbf{R}^3$ , respectively.

**SOLUTION:**

We can write it as

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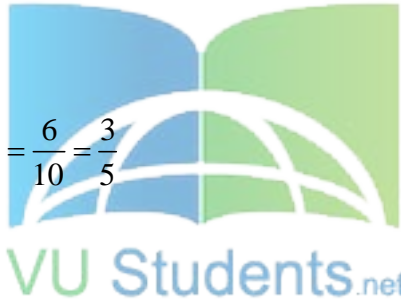
$$x = c_1 u_1 + c_2 u_2 + c_3 u_3$$

$$c_1 = \frac{x \cdot u_1}{u_1 \cdot u_1} = \frac{\begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}}{\begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}} = \frac{15+9+0}{9+9+0} = \frac{24}{18} = \frac{12}{9} = \frac{4}{3}$$

$$c_2 = \frac{x \cdot u_2}{u_2 \cdot u_2} = \frac{\begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}}{\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}} = \frac{10-6-1}{4+4+1} = \frac{3}{9} = \frac{1}{3}$$

$$c_3 = \frac{x \cdot u_3}{u_3 \cdot u_3} = \frac{\begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}}{\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}} = \frac{5-3+4}{1+1+8} = \frac{6}{10} = \frac{3}{5}$$

$$x = \frac{4}{3} u_1 + \frac{1}{3} u_2 + \frac{3}{5} u_3$$



$$\hat{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

5. Compute the least square error associated with the least square solution of the

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$$

equation  $Ax = b$  where

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6. Solve the following homogeneous system of linear equations:

$$x_1 + 3x_2 - x_3 = 0$$

$$x_2 - 8x_3 = 0$$

$$4x_3 = 0$$

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$$x(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

7. Solve the initial value problem  $x'(t) = Ax(t)$  for  $t \geq 0$ , with  $x(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ . Where

$$A = \begin{bmatrix} -2 & -5 \\ 1 & 4 \end{bmatrix},$$

$$v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

with eigenvectors

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8. Let  $V$  be an inner product space. Show that if  $u$  and  $v$  are orthogonal vectors in  $V$  such

$$\|u\| = \|v\| = 1 \quad \|u - v\| = \sqrt{2}$$

that , then .